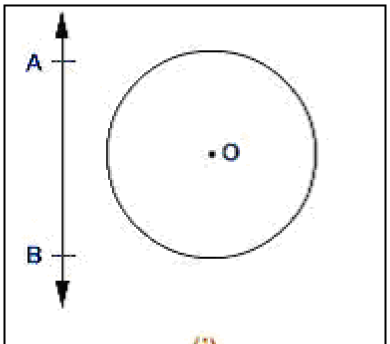
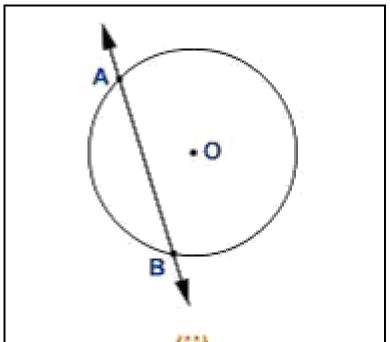
Circles

Tangent to a Circle

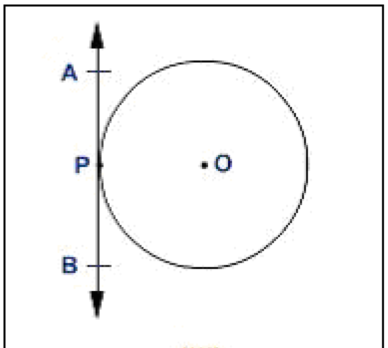
A tangent is a line touching a circle at one point



(i)



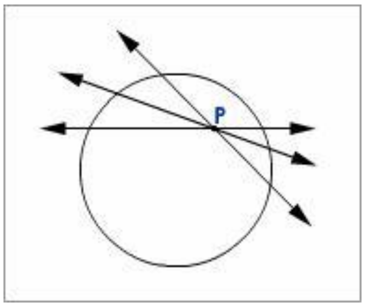
(ii)



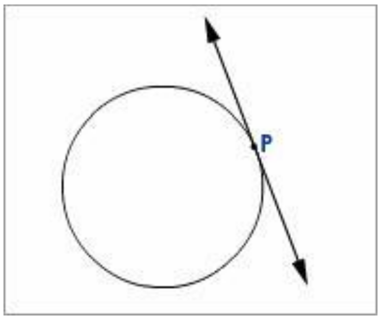
(iii)

1. **Non-intersecting line** - fig (i): The circle and the line AB have no common point.
2. **Secant** - fig (ii): The line AB intersects the circle at two points A and B. AB is the secant of the circle.
3. **Tangent** - fig (iii): The line AB touches the circle at only one point. P is the point on the line and on the circle. P is called the point of contact. AB is the tangent to the circle at P.

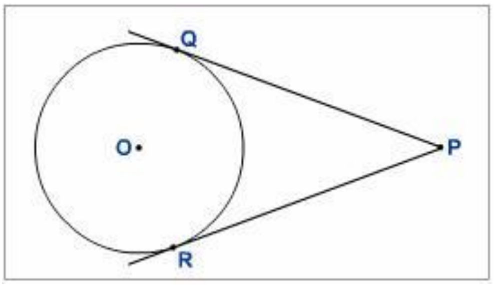
Number of Tangents from a Point on a Circle



From a point inside a circle, no tangents can be drawn to the circle.



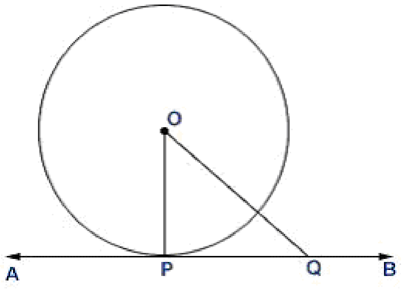
From a point on a circle, only 1 tangent can be drawn to the circle.
In this figure, P is a point on the circle. There is only 1 tangent at P. P is called the point of contact.



From a point outside a circle, exactly 2 tangents can be drawn to the circle. In this figure, P is the external point. PQ and PR are the tangents to the circle at points Q and R respectively. The length of a tangent is the length of the segment of the tangent from the external point to the point of contact. In this figure, PQ and PR are the lengths of the 2 tangents.

Theorem 1:

The tangent at any point of a circle is perpendicular to the radius through the point of contact.



Given:

AB is a tangent to the circle with centre O. P is the point of contact. OP is the radius of the circle.

To prove:

OP ⊥ AB

Proof:

Let Q be any point (other than P) on the tangent AB.

Then Q lies outside the circle.

$\Rightarrow OQ > r$

$\Rightarrow OQ > OP$ For any point Q on the tangent other than P.

$\Rightarrow$ OP is the shortest distance between the point O and the line AB.

$\Rightarrow$ OP ⊥ AB

(∵The shortest line segment drawn from a point to a given line, is perpendicular to the line)

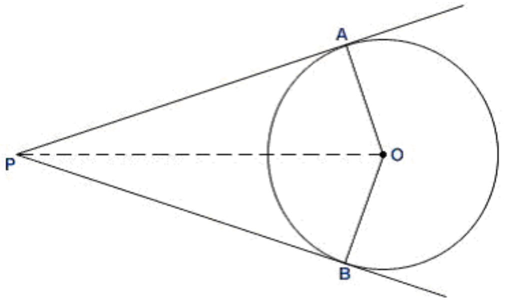
Thus, the theorem is proved.

From the above theorem,

1. The perpendicular drawn from the centre to the tangent of a circle passes through the point of contact.
2. If OP is a radius of a circle with centre O, a perpendicular drawn on OP at P, is the tangent to the circle at P.

Theorem2:

The lengths of tangents drawn from an external point to a circle are equal.



Given:

P is an external point to a circle with centre O. PA and PB are the tangents from P to the circle. A and B are the points of contact.

To prove:

PA = PB

Construction:

Join OA, OB, OP.

Proof:

In triangle APO and BPO,

Statement	Reason
OA = OB	Radii of the same circle
$\angle OAP = \angle OBP = 90°$	The radius is perpendicular to the tangent at the point of the contact.
OP = OP	Common
$\Delta OAP \cong \Delta OBP$	By SAS postulate
PA = PB	CPCT(Third side of the triangles)

From the above theorem,

1. $\angle AOP = \angle BOP$ (CPCT) This states that the two tangents subtend equal angles at the centre of the circle
2. $\angle APO = \angle BPO$ (CPCT) The tangents are equally inclined to the line joining the point and the centre of the circle.

   Or the centre of the circle lies on the angle bisector of the $\angle APB$.